Heat Transfer Modeling Aims

- Theoretical rate of heat production
- Calorimetric technique?
- Heat Transfer in liquid vs. solid growth media
- Analytical and computational methods

Theoretical Rate of Heat Production

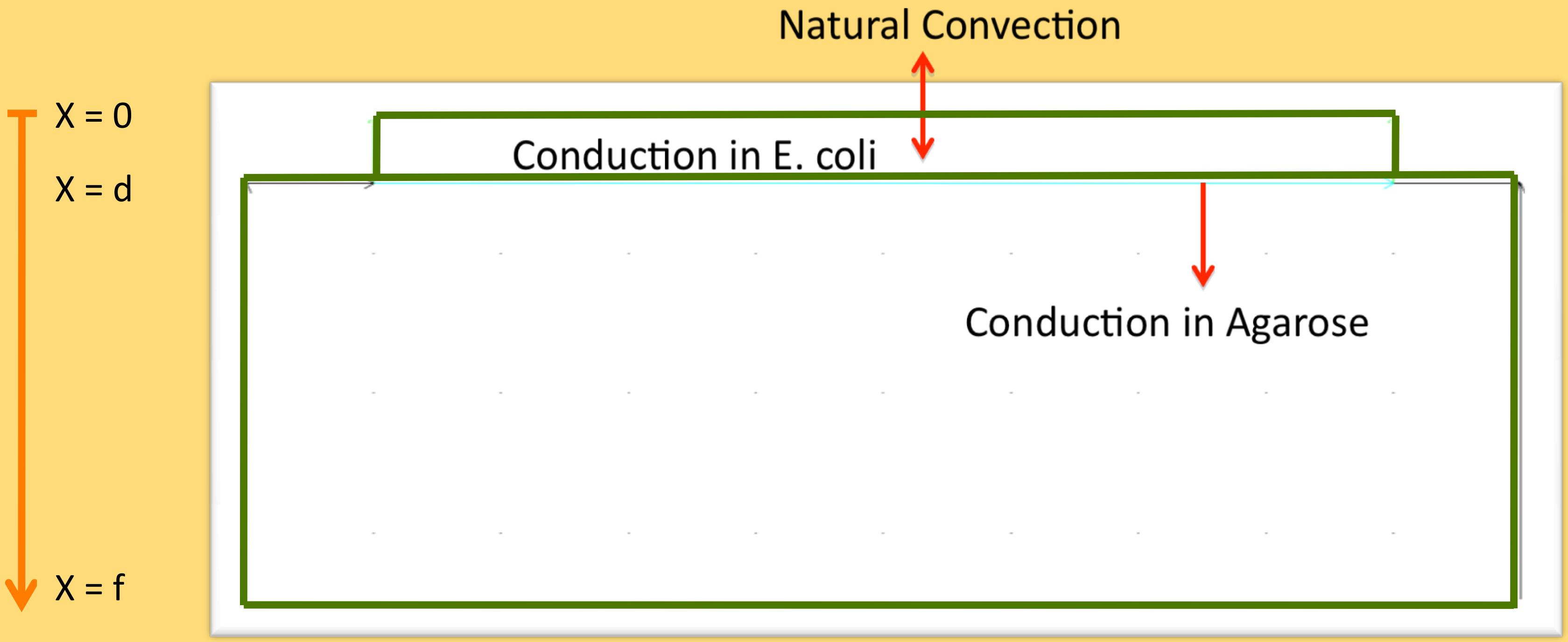
Via AOX Pathway

- 800 mV electric potential drop of 4 electrons generates 5.12 x 10^{-19} Joules
- 70% of electrons enter AOX pathway
- Assume time scale of ATP cycle to calculate power
- Power generated per cell is 1.6 x 10^{-13} Watt

Heat Transfer in Liquid Media

- Simplifying Assumptions
    1. Liquid solution can be assumed water
    2. Complete insulation
    3. Heat accumulation within system
    4. Homogeneous mixture
    5. No work done on or performed by the system
- Density of bacterial culture can vary by 2 orders of magnitude
- Temperature of system can be raised by 1K in 4 – 40 min.

Heat Transfer in Solid Growth Media



Assumptions:

1. Petri dish is completely insulated, and kept at 288K
2. Ambient temperature is 288K
3. Conduction through E. coli is similar to that in water
4. Constant coefficients for conductivity in both media, constant convective coefficient for air
5. Aspect ratio : width of colony >> height of colony

Steady-State Temperature Profile

1D Control-Volume (E. coli) using Rectangular Coordinates

Control-Volume 1: E. coli

- The general energy equation: No shaft or viscous work, no accumulation, steady-state

$$\cancel{\frac{\partial Q}{dt}} - \cancel{\frac{\partial W_s}{dt}} - \cancel{\frac{\partial W_\mu}{dt}} = \iint_{c.s.} \left(e + \frac{p}{\rho}\right)\rho(v \bullet n)\,dA + \frac{\partial}{dt}\iiint_{c.v.} e\rho\, dV$$

- Incompressible fluid, without velocity, constant k, 1-Dimensional heat transfer

$\alpha\nabla^2 T = -\frac{Q}{\rho c_p}$ Where $k = \alpha\rho c_p$ and $\nabla^2 T = \frac{d^2T}{dx^2} + \cancel{\frac{d^2T}{dy^2}} + \cancel{\frac{d^2T}{dz^2}}$

- Integrating Poisson Equation

$$\frac{d^2T}{dx^2} = -\frac{Q}{k} \rightarrow \frac{dT}{dx} = \frac{-Q}{k}x + C_1 \rightarrow \boxed{T(x)_{Ecoli} = \frac{-Q}{2k}x^2 + C_1x + C_2}$$

Steady-State Temperature Profile

1D Control Volume (Agarose) using Rectangular Coordinates

Control-Volume 2: Agarose

- The general energy equation is simplified to Laplace equation.
- Integrating Laplace Equation:

$$\frac{d^2T}{dx^2} = 0 \quad \rightarrow \quad \frac{dT}{dx} = C_3 \quad \rightarrow \quad \boxed{T(x)_{Agarose} = C_3 x + C_4}$$

- It is not a quadratic function of *X* : no heat generation term.

Analytical Solutions

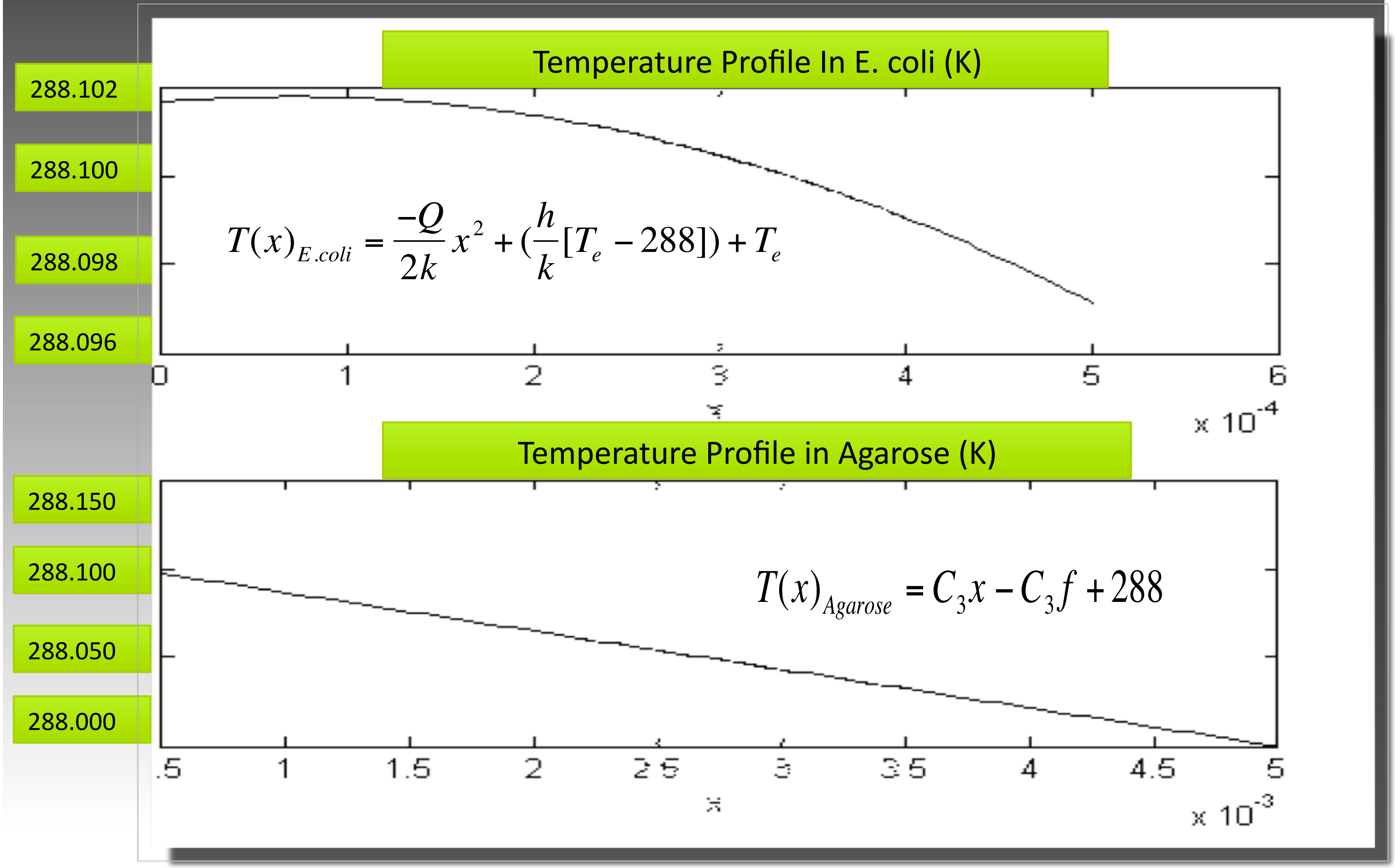
Solving for Boundary Conditions (BCs)

$$T(x)_{Ecoli} = \frac{-Q}{2k}x^2 + C_1x + C_2 \quad \longleftrightarrow \quad T(x)_{Agarose} = C_3x + C_4$$

- Solving for C_1 , C_2, C_3, and C_4 :
  - ✓ Q: volumetric flow of heat generated by E. coli
  - ✓ k : conductive coefficient of water at 288K
  - ✓ h : convective coefficient of air at 288K
  - ✓ $T_{ambient}$: 298 K
  - ✓ Measurements of height of colony and agarose
  - ○ $C_2 = T_e$, Temperature at Ecoli- air boundary (unknown)

- Heat flux at the E. coli - air boundary was equal to the convective heat flux (X = 0)
- Heat flux and temperature were equated at E. coli - agarose boundary (X = d)

Boundary Conditions were calculated: 0.1K change in temperature.



Temperature Profile In E. coli (K)

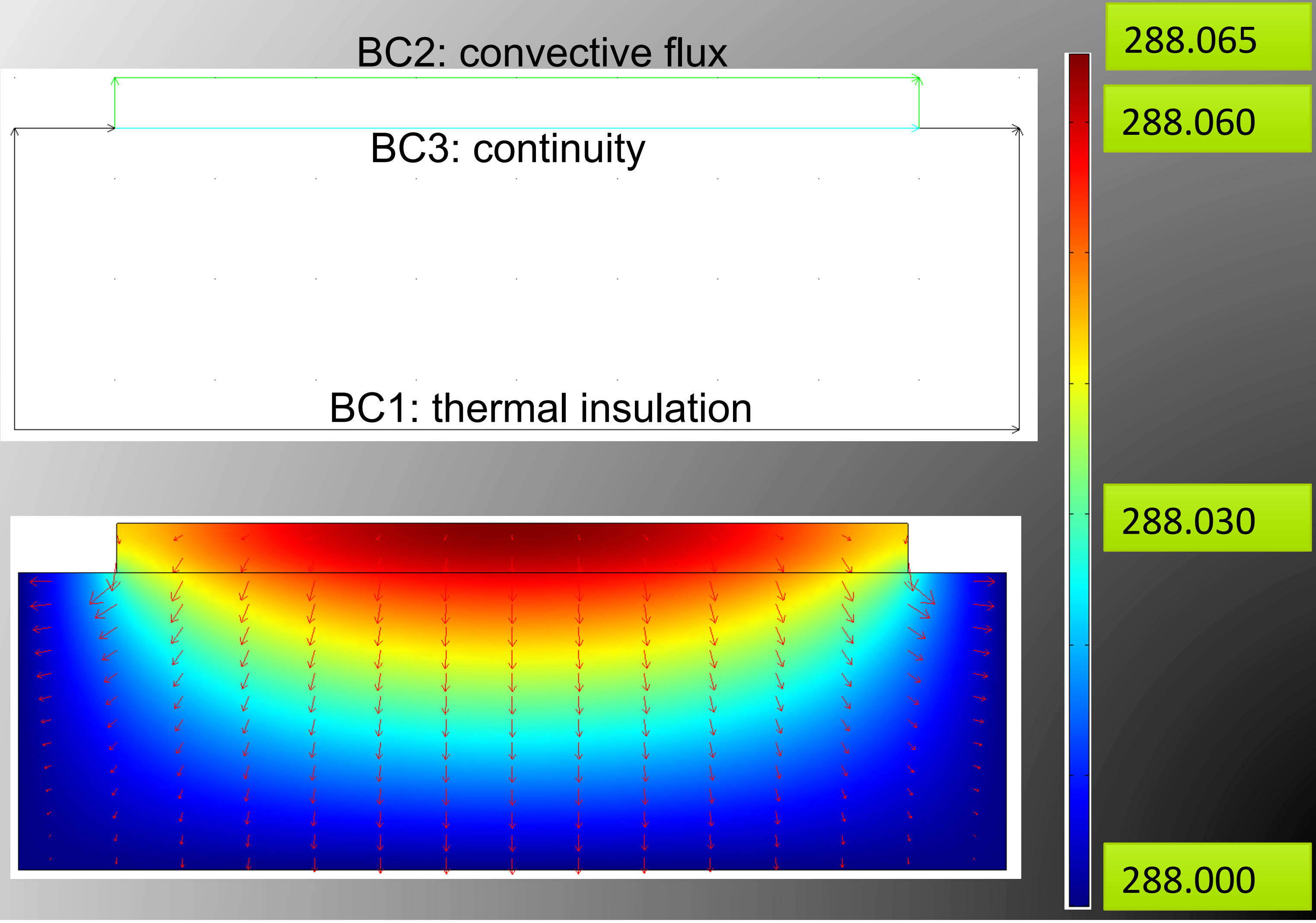
$$T(x)_{E.coli} = \frac{-Q}{2k}x^2 + (\frac{h}{k}[T_e - 288]) + T_e$$

Temperature Profile in Agarose (K)

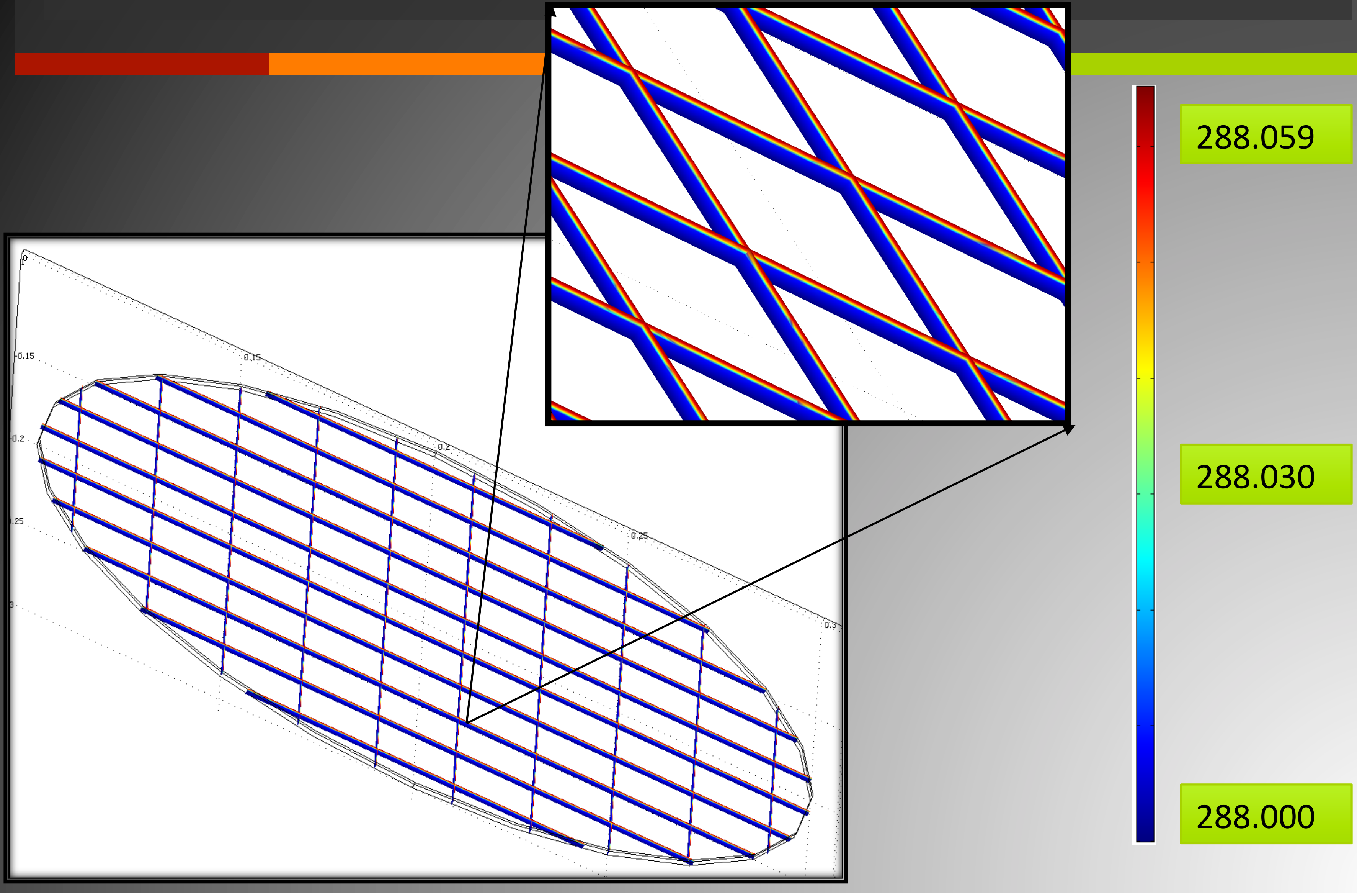
$$T(x)_{Agarose} = C_3 x - C_3 f + 288$$

Computational Approach:

2D Temperature Profile in COMSOL



3D Colony Model



Conclusions on Modeling

- Within solution 1K change in temperature in 4 – 40 minutes.
- On agar, steady state temperature profile derived analytically matches closely with those found computationally using COMSOL.
- Using 1D control-volume is a good assumption, since 3D temperature profile was not considerably different.
- Derived analytically and computationally, the change in temperature due to AOX expression should be approximately 0.1 K (on solid growth media).
- Due to better accumulation of energy in liquid media, characterization of heat production may be more accessible using a liquid culture.
- A highly sensitive (at least 0.1K) thermal imaging camera will be essential for measuring heat production of bacterial colony in both liquid and solid growth media.