Spatiotemporal Model of Toggle Switch Expression Circuit

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# 1 Qualitative Considerations

For the purposes of this simulation, we assume the regulatory effect of the cI/lacI toggle switch is perfect, and so as far as we are concerned, each cell is either in a state of low lacI or a state of high lacI. Those that have low lacI are incapable of producing AHL, luxI, luxR or mCherry, while those that have high lacI are incapable of producing GFP. The overall circuit that we are considering, then, is simply:

pLux/CI-OR: LuxI

pLux/CI-OR: LuxR

pLux/CI-OR: mCherry

pLux/lacO: GFP

AHL interacts with these compounds in a relatively straightforward manner. luxI acts as a catalyst for the production of AHL, and AHL promotes the activation of luxR. luxR, which in its native V. fischeri activates fluorescence, here merely acts as a transcription factor for pLux. Now, pLux/CI-OR leaks, i.e. is always on, at some constant background rate, though we assume here that even this leaking is turned off by the presence of cI. AHL is the only compound that diffuses between cells. Finally, we treat the plate of cells as a continuous distribution of cells, in which cell density $N$, as well as chemical concentrations, are functions of the two spatial variables and time.

# 2 System Dynamics

For low lacI cells:

$$\frac{d[AHL]}{dt} = D_1 \nabla^2 [AHL] + N k_{AHL}[luxI] - \gamma_{AHL}[AHL]$$

$$\frac{d[luxI]}{dt} = k_{leak}N - \gamma_{luxI}[luxI]$$

$$\frac{d[luxR]}{dt} = k_{leak}N - \gamma_{luxR}[luxR]$$

$$\frac{d[mCherry]}{dt} = k_{leak}N - \gamma_{mCherry}[mCherry]$$

$$\frac{d[GFP]}{dt} = k_{leak}N + \alpha_p N \frac{[luxR]^a [AHL]^b}{K_M + [luxR]^a [AHL]^b} - \gamma_{GFP}[GFP]$$

$$\frac{dN}{dt} = \alpha_N N (1 - \frac{N}{N_l})$$

For high lacI cells:

$$\frac{d[AHL]}{dt} = D_1 \nabla^2 [AHL] + N k_{AHL} [luxI] - \gamma_{AHL} [AHL]$$

$$\frac{d[luxI]}{dt} = k_{leak} N + \alpha_p N \frac{[luxR]^a [AHL]^b}{K_M + [luxR]^a [AHL]^b} - \gamma_{luxI} [luxI]$$

$$\frac{d[mCherry]}{dt} = k_{leak} N + \alpha_p N \frac{[luxR]^a [AHL]^b}{K_M + [luxR]^a [AHL]^b} - \gamma_{mCherry} [mCherry]$$

$$\frac{d[luxR]}{dt} = k_{leak} N + \alpha_p N \frac{[luxR]^a [AHL]^b}{K_M + [luxR]^a [AHL]^b} - \gamma_{luxR} [luxR]$$

$$\frac{d[GFP]}{dt} = k_{leak} N - \gamma_{GFP} [GFP]$$

$$\frac{dN}{dt} = \alpha_N N (1 - \frac{N}{N_l})$$

These dynamics show qualitatively correct results, and the paramaters need to be determined using experimental data. Further restraints do need to be imposed, however, such as reflecting diffusion off the boundaries (which can be built into the model by setting the appropriate spatial derivatives to 0), as well as removing the idealization of the binary state for [lacI]. The given system has been solved using the NDSolve command in Mathematica, and has been visualized in graphical form. It will be relatively straightforward to literally visualize this by having Mathematica display colors of different intensities to represent our lawn of cells.